Math 31 Formulae

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\sin\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

Addition and Subtraction Formulae

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$
$$\sin(\theta-\phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$$
$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$
$$\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$$
$$\tan(\theta+\phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$$
$$\tan(\theta-\phi) = \frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi}$$

Double Angle Formulae

$$\sin 2\theta = 2\sin\theta\cos\phi$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$
$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Squares of Sine and Cosine

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$
$$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

Half Angle Formulae

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Related and Co-related Angles

$$\sin(\pi-\theta) = \sin\theta$$
$$\cos(\pi-\theta) = -\cos\theta$$
$$\tan(\pi-\theta) = -\tan\theta$$
$$\sin(\pi+\theta) = -\sin\theta$$
$$\cos(\pi+\theta) = -\cos\theta$$
$$\tan(\pi+\theta) = \tan\theta$$
$$\sin(2\pi-\theta) = -\sin\theta$$
$$\cos(2\pi-\theta) = \cos\theta$$
$$\tan(2\pi-\theta) = -\tan\theta$$
$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$$
$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$
$$\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$$
$$\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$$
$$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$$
$$\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta$$

$$\sin\left(\frac{3\pi}{2}-\theta\right) = -\cos\theta$$
$$\cos\left(\frac{3\pi}{2}-\theta\right) = -\sin\theta$$
$$\tan\left(\frac{3\pi}{2}-\theta\right) = \cot\theta$$
$$\sin\left(\frac{3\pi}{2}+\theta\right) = -\cos\theta$$
$$\cos\left(\frac{3\pi}{2}+\theta\right) = \sin\theta$$
$$\tan\left(\frac{3\pi}{2}+\theta\right) = -\cot\theta$$

Fundamental Trigonometric Limit: $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin\theta = \cos\theta \qquad \frac{d}{dx}\cos\theta = -\sin\theta \qquad \frac{d}{dx}\tan\theta = \sec^2\theta$$

$$\frac{d}{dx}\csc\theta = -\csc\theta\cot\theta \qquad \frac{d}{dx}\sec\theta = \sec\theta\tan\theta \qquad \frac{d}{dx}\cot\theta = -\csc^2\theta$$

Domain Restrictions for 1 - 1 Trigonometric Functions

$$\sin^{-1}x = y \Leftrightarrow \sin y = x, -\frac{\pi}{2} \le y \le \frac{\pi}{2} \qquad \tan^{-1}x = y \Leftrightarrow \tan y = x, -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

$$\cos^{-1}x = y \Leftrightarrow \cos y = x, 0 \le y \le \pi$$

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

Sum of Series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{i=1}^{n} ar^{i-1} = \frac{a(1-r^n)}{1-r}$$

Table of Indefinate Integrals (Antiderivatives)

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \ (n \ne -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x\tan x\,dx = \sec x + C \qquad \int \csc x\cot x\,dx = -\csc x + C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a} + C \ (a \ne 1)$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$