Math 31 Unit 1 Exam

March 2015 Name _____

[marks]

1. Given the function, $f(x)=\frac{3}{2x-3}$,

a) find, (you may use a calculator) the slope of the **secant** which passes through the given points. In each case, one point is $(2, f(2))$. You do not need to show work.

i. $(2, f(2))$ and $(3, f(3))$

ii. $(2, f(2))$ and $(2.5, f(2.5))$

[4]

iii. $(2, f(2))$ and $(2.1, f(2.1))$

iv. $(2, f(2))$ and $(1.9, f(1.9))$

b) Using either your results from a) above, or **any other method** you like, determine the **slope** of the **tangent** to $f(x)=\frac{3}{2x-3}$ at the point $(2, f(2))$. A reasonable estimate will be acceptable, but **explain your reasoning** for how you arrived at your answer (exact or estimated).

[2]

2. Let $f(x)=\begin{cases} x^2-3, & x<-1 \\ 2x, & x\geq -1 \end{cases}, x\in R$

Find the following limits if they exist, or indicate why the limit does not exist.

a) $\lim_{x\to -1^-} f(x)$

b) $\lim_{x\to -1^+} f(x)$

[4]

c) $\lim_{x\to -1} f(x)$

d) Is the function $f(x)$ **discontinuous** anywhere in its domain?

3. Find $\lim_{n\to\infty}\left(\frac{12}{2^n}+\frac{5n^2-12n-4n^3}{32+2n^3-6n^2}\right)$

[2]

4. Find the sum of the **infinite** series ***or*** explain why this sum does not exist.

a) $9-12+16-\frac{64}{3}+\frac{256}{9}-\ldots$

[4]

b) $200-100+50-25+12.5-\ldots$

5) Determine, algebraically, and **showing work** $\lim_{x\to-3}\frac{x^2-x-12}{x^2-9}$

[3]

6 a). From basic principles, directly **from the definition of a derivative**, that is, using either

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \text{or} \quad f'(x)=\lim_{a\to x}\frac{f(x)-f(a)}{x-a}$$, find the derivative of

$$f(x)=\frac{2}{x+3}$$

[5]

6 b). From basic principles, directly **from the definition of a derivative**, that is, using either

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \quad \text{or} \quad f'(x)=\lim_{a\to x}\frac{f(x)-f(a)}{x-a}$$, find the derivative of

$f(x)=2-x^3$.

[5]

7. Differentiate the following functions using any of the applicable differentiation rules, and simplify:

a) $f(x)=\frac{25}{3\sqrt{x}}-\frac{2}{5x^2}$

[3]

b) $g(x)=(5x-2)^3\sqrt{x^2-12}$

[5]

7. (continued) Differentiate the given function using any of the applicable differentiation rules, and simplify:

c) $h(x)=\left(\dfrac{6x+5}{3x^2+2}\right)^4$

[5]

d) $m(x)=\dfrac{\sqrt{x^2+1}}{(2x^3-8)^5}$

[5]

8. A curve is defined by the equation $2x^2y^3 = 5x^3 + 2y + 5$

a) Using implicit differentiation, find $\frac{dy}{dx}$

[5]

9. Find both the first derivative, $\frac{dy}{dx}$, and the second derivative, $\frac{d^2y}{dx^2}$, of the function given by

$$y = 6x^5 - 2x^3 + x^2 + 12x + 15 - \frac{1}{3x}$$

[5]

10. Given the function $f(x)=\frac{15}{\sqrt{2x-5}}$

a) Determine the equation of the tangent line to the above curve at $(15,3)$

[7]

b) Find the point on $f(x)$ where the tangent line is parallel to the line $y=-\frac{5}{9}x$

[64] total marks