Math 31 - Unit 2 Exam

Name

6 April 2009 [marks]

Show all your work for all questions, explaining your method where necessary. Leave all answers in exact form (in terms of radicals, π , fractions or terminated decimals). No marks will be awarded for approximate answers. Your methods, procedures and explanations will be evaluated. Consider all given numbers to be exact.

1. Landon convinces president Obama to invest in Whillanswheels instead of throwing good money after bad bailing out GM! Landon uses the money to invest in a sophisticated robot which spray paints his super efficient automobiles. The nozzle of the sprayer moves according to the formula $s(t)=4t^3-33t^2+84t+5$ for the first 5 seconds ($0 \le t \le 5$) (t in seconds and s in cm). a) Determine the time(s) at which the nozzle is at rest (the velocity of the nozzle is $0 \ cm/s$)

b) Determine when (if at all) the acceleration of the nozzle is 0 cm/s^2

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c) Calculate the total **distance** that the nozzle travels in the first 5 seconds.

2. Annette watches in horror as a long ladder ($6\frac{1}{2}m$ long, to be exact) leaning against a vertical wall starts to slide away from the wall on a horizontal icy surface. A person is on the top rung of the ladder! How fast is the top of the ladder sliding down the wall when it is $2\frac{1}{2}m$ from the surface of the ice if the bottom is sliding away from the wall at 2m/s?

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(validating physics question: is this faster than freefall? - no marks if you answer this question)

3. Dissatisfied with environment Canada's weather forecasts, Tim resolves to improve the weather forecasts for the Peace country. To get upper atmosphere wind and temperature readings he obtains a weather balloon which he fills with helium. The helium is pumped into the balloon at the rate of

 $\frac{2}{15}m^3/min$ At what rate is the radius of the balloon increasing when the radius is $\frac{4}{5}m$ (80 cm)?

(show all work and write your give answer in exact form)

Note: the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$

4. Ashlee's new kite, the Petkite, (which even has a small propeller powered by a solar collector to improve maneuverability!) promises to be a big seller for the summer of 2010. She has determined that the cost function for her kite is C(x)=5000+3x+.0002x², with x being the number of Petkites produced and C the cost, in dollars. The demand function, in dollars, is p(x)=8-.00001x. So the revenue is given by R(x)=10x-.0003x². The profit, P, equals the revenue minus the cost. (remember that all numbers are exact) a) Determine the profit function in simplified form.

b) Determine the marginal profit function (the derivative of the profit function).

c) How many Petkites will Ashlee produce in order to maximize her profit?

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d) What will her profit be?

5. As a classical pianist, Stephanie travels a lot but must protect her ears. She needs to walk between two noisy jets, one 3 times as loud as the other, to get to her small plane. The jets are 80 m apart. Consider that the jets are point sources of noise and that the loudness of the sound (L) at Stephanie's ears is inversely proportional to the intensity of the sound (I) of the source and inversely

proportional to the distance (d) she is away from the sound. $L = \frac{I}{d^2}$. At what point, between

the two jets, should she walk to minimize the loudness she experiences as she walks between the jets? You may give the distance from either jet, but indicate from which jet you are giving the distance. Leave your answer in exact form.

- 6. Given the function $f(x) = \frac{4 x^2}{1 + x^2}$,
 - a) Determine the y intercept, or, if it does not exist, explain why it does not exist.

b) Determine all existing *x* intercepts.

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c) Give any horizontal asymptotes, or, if none exist, explain why.

d) Determine if the graph of this function has even or odd symmetry, or neither. Show your work!

- 6. (continued) We were examining the function $f(x) = \frac{4 x^2}{1 + x^2}$,
 - d) Determine the intervals of increase and the intervals of decrease of f(x).

[5]

e) determine the intervals of upward and downward concavity.

[5]

- 7. Given the function $g(x) = \frac{4}{x^3} \frac{3}{x}$,
 - a) Determine if g(x) even, odd symmetry or neither. (show work)

b) Give the equation of all the vertical and horizontal asymptotes.

[8]

c) Determine the coordinates (both *x* and *y* coordinates) of all the maximum and minimum values and indicate whether each is a *local or absolute* and *maximum or minimum*. Show work!

d) Determine the coordinates (both x and y coordinates) of all the points of concavity of g(x). (show work!)

[3]