Math 31 - Unit 2 Exam Part A

March 2014 Name ______________________________

[marks]

1. Unbalanced forces act on a particle so that it moves according to the function $s(t)=4t^3-31t^2+70t$ while $0 \le t \le 5$. t is measured in seconds and s is measured in cm.

a) What is the particle's **velocity** after 2.0 s? (give an exact answer)

$$v(t)=s'(t)=12t^2-62t+70$$
$$v(2)=12(2)^2-62(t)+70=-6$$

The velocity equals $-6\,cm/s$ at 2.0 *s*

b) At what time(s), t , measured in seconds, is the particle at rest? (answers must be exact)

$$0=12t^2-62t+70$$
$$0=2(6t^2-31t+35)$$
$$0=2(2t-7)(3t-5)$$

so $2\mathrm{t}=7$ or $3t=5$

and $t=7/2\,s$ or $5/3\,s$

The particle is at rest at 7/2 *s* and at 5/3 *s*

c) What is the particle's acceleration after 2.0 s?

$$a(t)=v'(t)=s''(t)=24t-62$$

so $a(2)=24(2)-62=-14\,cm/s^2$

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d) When is the particle's **acceleration** equal to $0\ cm/s^2$?

$$0=24t-62$$
$$24t=62$$
$$t=\frac{62}{24}s=\frac{31}{12}s$$

e) Over what time period is the particle moving backwards (in a negative direction)?

The particle moves in a negative direction between 5/3 and 7/2 *s*.

2. Rena, who is 1.65 *m* tall, is walking away from a lamppost at 1.2 *m/*s. The light at the top of the lamppost is 4.95 m above the ground. How fast is Rena's shadow lengthening when she is 12 m from the lamppost?

let x be the distance Rena is from the lamppost and y the length of her shadow, then

[6] by the proportions of similar triangles $\frac{x+y}{y}=\frac{4.95}{1.65}=3$ and $x+y=3y$ or $y=\frac{1}{2}x$

then $\frac{dy}{dt}=\frac{1}{2}\frac{dx}{dt}$, We know that $\frac{dx}{dt}=1.2\,m/s$, so $\frac{dy}{dt}=\frac{1}{2}1.2\,m/s=0.6\,m/s$

Rena's shadow is lengthening by 0.6 *m/s*. (This result is independent of the how far Rena has walked.)

3. Grant is using a large conical tank to hold rainwater for his greenhouse. The tank has a diameter of 2.0 *m* at the top and is 2.5 *m* high. During a June shower, rain is flowing into the tank at the rate of 0.5 *l/s* (litres per second). Oh, yes, that's $500\,cm^3/s$ How fast is the rainwater rising in the tank when the water level in the tank has reached a height of $80\,cm$ from the bottom?

$\left(\text{Note: the volume of a cone is given by } V=\frac{1}{3}\pi r^2 h\right)$

$$\frac{h}{r}=\frac{2.5}{1} \text{ and } r=\frac{h}{2.5} \text{ so } V=\frac{1}{3}\pi\left(\frac{h}{2.5}\right)^2 h$$

$$V=\frac{\pi}{18.75}h^3$$

$$\frac{dV}{dt}=\frac{\pi}{18.75}3h^2\frac{dh}{dt}$$

$$=\frac{3\pi}{18.75}h^2\frac{dh}{dt}$$

[6]

now solving for $\frac{dh}{dt}$, we get

$$\frac{dh}{dt}=\frac{dV}{dt}\frac{18.75}{3\pi h^2}$$

substituting

$$\frac{dh}{dt}=500\frac{18.75}{3\pi 80^2}=\frac{125}{256\pi}\,cm/s$$

The water is rising at $\frac{125}{256\pi}\,cm/s$ (aboout 9.33 *cm* per minute)

4. Logan plans to start a business selling subs. After doing his research, he determines that the cost function for *Logan's Subs* is given by $C(n) = 8200 + 1.3n$ Where C is the cost in Canadian Dollars and n is the number of submarine produced. The demand function, in dollars, is $p(n) = 5.2 - 0.0001n$.

a) Determine the **revenue function**. Note: the revenue function is the product of the **demand** (or price) **function** and **quantity** of subs sold.

$$R(n) = 5.2n - 0.0001n^2$$

b) Find the **profit function** (the revenue minus the cost) in simplified form.

$$\begin{aligned} P(n) &= R(n) - C(n) \\ &= 5.2n - 0.0001n^2 - 8200 - 1.3n \\ &= 3.9n - 0.0001n^2 - 8200 \end{aligned}$$

c) Determine the **marginal profit** function (the derivative of the profit function).

[5] $P'(n) = 3.9 - 0.0002n$

d) How many submarines should Logan produce so as to maximize his profit?

For critical values, the derivative vanishes or DNE, so

$$\begin{aligned} 0 &= 3.9 - 0.0002n \\ 3.9 &= 0.0002n \\ n &= 195000 \end{aligned}$$

Logan should produce 195 000 subs to maximize his profit.

e) What will the profit be if the profit is maximized?

His profit, $P(195000)$, will be \$29825.00

5. Delaney has a square piece of cardboard $80\,cm$ by $80\,cm$ that he wishes to make into a box of **maximum volume**. He has plenty of tape, a square a tape measure and a utility knife. He intends to cut out **squares from each of the corner** of the piece of cardboard and fold up the flaps and use tape to keep the box together. He does not need a top to the box. How large should his cut out squares be? (for maximum volume? $V=l\,w\,h$)

Let x be the length of the square Delaney cuts out of each corner. Then the dimensions of the box will be ($80 - 2x$) by ($80 - 2x$) by x. The volume of his box is $V=(80-2x)^2 x$. He takes the derivative of the volume with respect to x and sets that equal to 0 for the possible extreme values, as follows:

$$\frac{dV}{dx}=2(80-2x)(-2)x+(80-2x)^2 \text{ and so}$$
$$0=-4x(80-2x)+(80-2x)^2$$
$$0=(80-2x)[-4x+(80-2x)]$$
[6] $$0=(80-2x)(80-6x) \text{ so either}$$
$$0=80-2x \text{ or } 0=80-6x$$

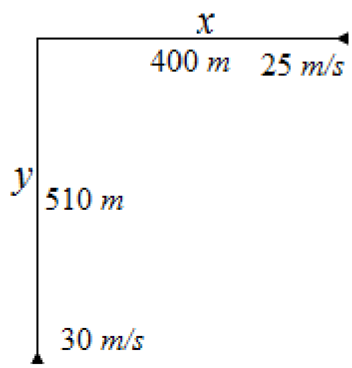
now if $80-2x=0$ then $x=40$ and there is no cardboard left!

so $80-6x=0$ and $x=\frac{40}{3}=13\frac{1}{3}$

$6x=80$

Delaney cuts out $13\frac{1}{3}\,cm$ squares from each corner to make is box.

6. At exactly 12:15:00 pm, Deanna is driving her car **west** at exactly **25 *m/s*** (90 *km/h*). She is **approaching** an unmarked intersection which is still exactly **400 *m* in front of** her. Nick, at exactly the same instant, is driving his car **north** and **approaching the same** unmarked intersection. He is still **510 *m*** from the intersection and is travelling at exactly **30 *m/s*** (108 *km/h*). If both Deanna and Nick maintain their speeds exactly, how many seconds after 11:15:00 pm do Deanna and Nick get as close to each other as they get? **Note: your answer must be exact!** Clearly show your method (calculus) and reasoning, including a well marked diagram.



Treating the intersection of the roads as the origin of a coordinate system, and positive directions as east and north, with time (*t* - in seconds) set at 0 at exactly 12:15:00 pm, we can specify the locations of Deanna (*x*) and Nick (*y*) - both in *m* – as follows:

[7] $x=400-25t$ and $y=-510+30t$

The distance (*d*) between them is then given by

$$d=\sqrt{(400-25t)^2+(-510+30t)^2}$$

To find the minimum, we take the derivative and set it to 0

$$\frac{dd}{dt}=\frac{1}{2}\left((400-25t)^2+(-510+30t)^2\right)^{-\frac{1}{2}}(2)(400-25t)(-25)+(2)(-510+30t)(30)$$

$$=\frac{2(400-25t)(-25)+2(-510+30t)(30)}{\sqrt{(400-25t)^2+(-510+30t)^2}}$$

The denominator will only be positive or 0 if the distance is 0. We can verify that this does not happen. So, for the minimum, we have

$$0=(400-25t)(-25)+(-510+30t)(30)$$
$$=(80-5t)(-5)+(-102+6t)(6)$$
$$=-400+25t-612+36t$$
$$1012=61t$$
$$t=\frac{1012}{61}\,s$$

So, after about 16.59 *s* they get to about 19.2 *m* apart.

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[40] total marks