

Math 31 - Unit 2 Exam Part A

March 2014

Name \_\_\_\_\_

[ marks ]

1. Unbalanced forces act on a particle so that it moves according to the function

$s(t) = 4t^3 - 31t^2 + 70t$  while  $0 \leq t \leq 5$ .  $t$  is measured in seconds and  $s$  is measured in cm.

a) What is the particle's **velocity** after 2.0 s? (give an exact answer)

b) At what time(s),  $t$ , measured in seconds, is the particle at rest? (answers must be exact)

c) What is the particle's acceleration after 2.0 s?

[ 10 ]

d) When is the particle's **acceleration** equal to  $0 \text{ cm/s}^2$ ?

e) Over what time period is the particle moving backwards (in a negative direction)?

2. Rena, who is  $1.65\text{ m}$  tall, is walking away from a lamppost at  $1.2\text{ m/s}$ . The light at the top of the lamppost is  $4.95\text{ m}$  above the ground. How fast is Rena's shadow lengthening when she is  $12\text{ m}$  from the lamppost?

[ 6 ]

3. Grant is using a large conical tank to hold rainwater for his greenhouse. The tank has a diameter of  $2.0\text{ m}$  at the top and is  $2.5\text{ m}$  high. During a June shower, rain is flowing into the tank at the rate of  $0.5\text{ l/s}$  (litres per second). Oh, yes, that's  $500\text{ cm}^3/\text{s}$ . How fast is the rainwater rising in the tank when the water level in the tank has reached a height of  $80\text{ cm}$  from the bottom?

$\left( \text{Note: the volume of a cone is given by } V = \frac{1}{3}\pi r^2 h \right)$

[ 6 ]

4. Logan plans to start a business selling subs. After doing his research, he determines that the cost function for *Logan's Subs* is given by  $C(n) = 8200 + 1.3n$  Where  $C$  is the cost in Canadian Dollars and  $n$  is the number of submarine produced. The demand function, in dollars, is  $p(n) = 5.2 - 0.0001n$ .

a) Determine the **revenue function**. Note: the revenue function is the product of the **demand** (or price) **function** and **quantity** of subs sold.

b) Find the **profit function** (the revenue minus the cost) in simplified form.

c) Determine the **marginal profit** function (the derivative of the profit function).

[ 5 ]

d) How many submarines should Logan produce so as to maximize his profit?

e) What will the profit be if the profit is maximized?

5. Delaney has a square piece of cardboard  $80\text{ cm}$  by  $80\text{ cm}$  that he wishes to make into a box of **maximum volume**. He has plenty of tape, a square a tape measure and a utility knife. He intends to cut out **squares from each of the corner** of the piece of cardboard and fold up the flaps and use tape to keep the box together. He does not need a top to the box. How large should his cut out squares be? (for maximum volume?  $V = lwh$  )

6. At exactly 12:15:00 pm, Deanna is driving her car **west** at exactly **25 m/s** (90 km/h). She is **approaching** an unmarked intersection which is still exactly **400 m in front of** her. Nick, at exactly the same instant, is driving his car **north** and **approaching the same** unmarked intersection. He is still **510 m** from the intersection and is travelling at exactly **30 m/s** (108 km/h). If both Deanna and Nick maintain their speeds exactly, how many seconds after 12:15:00 pm do Deanna and Nick get as close to each other as they get?
- Note: your answer must be exact!** Clearly show your method (calculus) and reasoning, including a well marked diagram.

[ 7 ]