Math 31 - Unit 2 Exam Part A

2 April 2019 Name _____

[marks]

1. Rom's tricky drone moves according to the function $\vec{s}(t)=2t^3-21t^2+60t+5$ for when $0\le t\le 6$ with t being measured in seconds and $\vec{s}$ in cm.

   a) What is Rom's drone's **instantaneous velocity** at 4 s?
   (show work and give an exact answer)

   b) At what time(s), t, is the drone's velocity 0 m/s?
   (show work and give an exact answer)

   c) What is the drone's **instantaneous acceleration** at 3 s? (show work!)

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   d) When does the drone's **acceleration** equal $0\,m/s^2$?

   e) Over what time period is the drone moving backwards (in a negative direction)?

   f) What is the total **distance** the drone moves between 0 and 6 s?

For all remaining questions, include a labelled diagram when appropriate, define your variables, explain your sign convention (positive or negative) as needed, show all work, explaining when and why you might be taking derivatives and answer in a sentence! Take all numbers to be **exact** (infinite significant digits) and give your answer as an **exact** number with units.

2. Mackenzie is using a pyramidal tank (inverted – with the point at the bottom) to collect rainwater. The top is a square with a side length, $\vec{s}$, of $2m$ and with a height, h , of $0.8m$. The capacity of the tank (the volume of water it can take when full) is given by the formula $V=\frac{1}{3}s^2h$. If rainwater is entering the tank at a rate of $5litres/min$ (equivalent to $0.005m^3/min$) at what rate is the water level inside the tank rising when the tank is filled up to a height of $0.6m$?

[6]

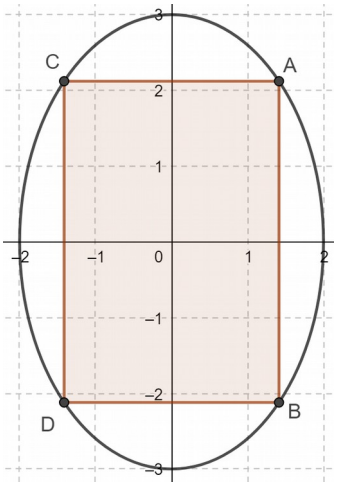
3. Kade is 100 *m* west of the intersection of 108 Avenue and 109 Street in Fairview. He is walking west (**away from** the indicated intersection) at 1.65 *m/s*. At the same instant, Reid is 125 *m* south of the same intersection and walking north (**toward** the above mentioned intersection) at 1.80 *m/s*. How fast is the direct distance between Kade and Reid changing at that instant? Also, indicate whether the distance is increasing or decreasing.

[6]

4. Ryo, who is $1.83\ m$ tall (or so be claims), is walking **away** from a lamppost at $1.58\ m/s$. The light at the top of the lamppost is $3.66\ m$ above the ground. How fast is Ryo's shadow lengthening when he is $14\ m$ from the lamppost?

[6]

5. Levi is making plans for landscaping an area of his yard. He is working with an elliptical area for a flower bed and shrubs. The curve of the ellipse is as shown in the diagram at the right and has the equation $9x^2+4y^2=36$ (The units are in metres). Solving this equation for y results in the equations $y=\pm\frac{\sqrt{36-9x^2}}{2}$. Levi hopes of have a raised rectangular bed inside the ellipse. Determine the **dimensions and area** of a rectangle inscribed in the ellipse (as in rectangle ABCD in the diagram shown) that has the largest possible area for such an inscribed rectangle. Note that the general coordinates of all the vertices of the given rectangle can easily be determined. For example, the coordinates of point A are $\left(x, \frac{\sqrt{36-9x^2}}{2}\right)$.



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[6]

6. Knight is interested in determining what the dimensions of a $2\,L$ cylindrical can should be in order to minimize its surface area and thus its cost. He uses the formulas for the volume and surface area of a cylinder as given here :

   $V = \pi r^2 h$ and $A = 2\pi r^2 + 2\pi r h$

   $V =$ volume, $A =$ surface area, $r =$ radius, and $h =$ height

   He also verifies that $2\,L$ equals $2000\,cm^3$. Using this data, determine the radius of such a cylindrical can that will minimize its volume. Give an **exact** answer.

[6]

7. Brooke travels a lot on her private plane. On a given day, she needs to walk between two noisy jets, one 3 times as loud as the other, to get to her small plane. The jets are $80\ m$ apart. Consider that the jets are point sources of sound and that the loudness of the sound (L) at Brooke's ears is directly proportional to the intensity of the sound (I) of the source and inversely proportional to the square of the distance (d) she is away from the sound: $\left(L=\frac{I}{d^2}\right)$. At what point, between the two jets, should Brooke walk to minimize the loudness she experiences as she walks between the jets? You may give the distance from either jet, but indicate from which jet you are giving the distance. Leave your answer in exact form.

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[48] total marks